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Grand unified textures for neutrino and quark mixings

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Abstract: The atmospheric neutrino data imply large mixing between the ν_μ and ν_τ states, $\theta_{23} = (45 \pm 12)^\circ$, while the MSW solution to the solar neutrino problem needs very small mixing angle $\theta_{12} = (2 \pm 1)^\circ$. In the quark sector the situation is rather opposite – the 23 mixing is tiny, $\theta_{23} \simeq 2^\circ$, versus reasonable 12 mixing, $\theta_{12} \simeq 13^\circ$. We show that such complementary patterns of the quark and leptonic mixings could naturally emerge in the context of the SU(5) grand unification, assuming that the fermion mass matrices have the Fritzsch-like structures but their off-diagonal entries are not necessarily symmetric. Such a picture exhibits a 'see-saw' like correspondence between the quark and leptonic mixing patterns so that the smaller the quark mixing angle is, the larger the corresponding leptonic mixing angle becomes. This fact simply follows from the fermion multiplet structure in SU(5). We also discuss a model with horizontal symmetry U(2) in which the discussed pattern of the mass matrices can emerge rather naturally.

Keywords: Solar and Atmospheric Neutrinos, Quark Masses and SM Parameters, GUT, Supersymmetric Standard Model.

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Contents

1. Introduction

Signals of neutrino oscillations accumulated during past several years impose strong constraints on the mass and mixing pattern of the three known neutrinos $\nu_{e,\mu,\tau}$. In particular, the atmospheric neutrino (AN) anomaly, a long-standing discrepancy by almost a factor of 2 between the predicted and observed ν_μ/ν_e ratio of the atmospheric neutrino fluxes, has recently received a strong confirmation from a high statistics experiment by the SuperKamiokande Collaboration [1]. These data indicate that the zenith angle/energy dependence of the atmospheric ν_μ flux is compatible with $\nu_\mu - \nu_\tau$ oscillation within the following parameter range:

$$\begin{aligned} \delta m^2_{\rm atm} &= (0.5-6)\times 10^{-3}\ {\rm eV}^2, \\ \sin^2 2\theta_{\rm atm} &> 0.82 \end{aligned} \tag{1.1}$$

(the best-fit values are $\delta m^2_{\rm atm} = 2.2\times 10^{-3}$ eV^2 and $\sin^2 2\theta_{\rm atm} = 1.0$), and disfavour the $\nu_\mu - \nu_e$ oscillation as a dominant reason for the AN anomaly.

On the other hand, the solar neutrino (SN) problem, an energy dependent deficit of the solar ν_e fluxes indicated by the solar neutrino experiments cannot be explained by nuclear or astrophysical reasons [2]. The most natural solution is provided by the resonant MSW oscillation [3] of ν_e into ν_μ, ν_τ or their mixture, which requires the following parameter range [4]:

$$\begin{aligned} \delta m^2_{\rm sol} &= (3-10)\times 10^{-6}\ {\rm eV}^2, \\ \sin^2 2\theta_{\rm sol} &= (0.1-1.5)\cdot 10^{-2} \end{aligned} \tag{1.2}$$

(the best-fit values are $\delta m^2_{\rm sol} = 5\times 10^{-6}$ eV^2 and $\sin^2 2\theta_{\rm sol} = 6\times 10^{-3}$).[1]

[1] The long wavelength *Just-so* oscillation from the Sun to the Earth [5] provides as good a fit as that of MSW, with the parameter range $\delta m^2_{\rm sol} \sim 10^{-10}$ eV^2 and $\sin^2 2\theta_{\rm sol} \sim 1$. However, in this paper we mainly concentrate on the pattern dictated by the MSW solution.

The explanation of the fermion mass and mixing pattern is beyond the capacities of the standard model (SM) and the neutrino case represents a part of the flavour problem. The masses of the charged fermions $q_i = (u_i, d_i)$, u_i^c, d_i^c; $l_i = (\nu_i, e_i)$, e_i^c ($i = 1, 2, 3$ is a family index) emerge from the Yukawa terms:

$$\phi_2 u_i^c \mathbf{Y}_u^{ij} q_j + \phi_1 d_i^c \mathbf{Y}_d^{ij} q_j + \phi_1 e_i^c \mathbf{Y}_e^{ij} l_j \,, \tag{1.3}$$

where $\phi_{1,2}$ are the Higgs doublets: $\langle\phi_{1,2}\rangle = v_{1,2}$, $(v_1^2 + v_2^2)^{1/2} = v_w = 174$ GeV and $\mathbf{Y}_{u,d,e}$ are general complex 3×3 matrices of the coupling constants.[2] In order to identify the physical basis of the fermion mass eigenstates, the Yukawa matrices $\mathbf{Y}_{u,d,e}$ and thus the fermion mass matrices should be brought to the diagonal form via the bi-unitary transformations:

$$\begin{aligned} U_u'^T \mathbf{Y}_u U_u &= \mathbf{Y}_u^D = \mathrm{Diag}(Y_u,\, Y_c,\, Y_t)\,, \\ U_d'^T \mathbf{Y}_d U_d &= \mathbf{Y}_d^D = \mathrm{Diag}(Y_d,\, Y_s,\, Y_b)\,, \\ U_e'^T \mathbf{Y}_e U_e &= \mathbf{Y}_e^D = \mathrm{Diag}(Y_e,\, Y_\mu,\, Y_\tau)\,. \end{aligned} \tag{1.4}$$

Hence, the quark mass eigenstates mix in the charged current $\bar{u}_i \gamma^\mu (1+\gamma_5) V_q^{ij} d_j$, and the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_q = U_u^\dagger U_d$ can be parametrized as [6]:

$$V_q = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.5}$$

where s_{ij} and c_{ij} respectively stand for the sines and cosines of three mixing angles θ_{12}, θ_{23} and θ_{13}, and δ is the CP-violating phase.

As for neutrino masses, they emerge only from the higher order effective operator [7]:

$$\frac{\phi_2\phi_2}{M}\, l_i \mathbf{Y}_\nu^{ij} l_j \,, \qquad \mathbf{Y}_\nu^{ij} = \mathbf{Y}_\nu^{ji} \,, \tag{1.6}$$

where $M \gg v_w$ is some cutoff scale and $\mathbf{Y}_\nu$ is the (symmetric) matrix of the dimensionless coupling constants. Thus, while the charged fermion masses are linear with respect to the weak scale v_w, the neutrino masses are bilinear which makes magnitudes of the latter naturally small.[3] One can go to the neutrino physical basis $\nu_{1,2,3}$ by the unitary transformation

$$U_\nu^T \mathbf{Y}_\nu U_\nu = \mathbf{Y}_\nu^D = \mathrm{Diag}(Y_1,\, Y_2,\, Y_3) \tag{1.7}$$

[2]Rather spontaneously we have chosen our notations as the ones adopted in the supersymmetric SM which in the following will be mentioned simply as SM, while the ordinary standard model, if any, we shall recall as a non-supersymmetric SM. Barring numerical details, all the main arguments put forward in our paper will be valid also for non-supersymmetric case.

[3]Any known mechanism for the neutrino masses effectively reduces to the operator (1.6) after integrating out the relevant heavy states (e.g. in the 'seesaw' mechanism, after integrating out the right-handed Majorana neutrinos with masses $\sim M$).

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and the mixing matrix $\tilde{V}_l = U_e^\dagger U_\nu$ in the leptonic current $\bar{e}_i\gamma^\mu(1+\gamma_5)V_l^{ij}\nu_j$ can be presented as:

$$\tilde{V}_l = V_l P_\nu = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix} , \tag{1.8}$$

where the first factor V_l can be parametrized in a manner similar to (1.5). It relates the neutrino flavour eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ to the mass eigenstates (ν_1, ν_2, ν_3), and describes the neutrino oscillation phenomena. (Due to the Majorana nature of neutrinos, the matrix $\tilde{V}_l$ contains two additional phases $\delta_{2,3}$, but these are not relevant for neutrino oscillations). In the following, we distinguish the quark and lepton mixing angles in V_q and V_l by the subscripts 'q' and 'l', respectively.

The mass spectrum of the quarks and charged leptons is spread over five orders of magnitude, from MeVs to hundreds of GeVs, with a strong inter-family hierarchy [8]:[4]

$$\begin{array}{lll} m_t = 163 \pm 5 \text{ GeV}, & m_c = 1.1 - 1.4 \text{ GeV}, & m_u = 2 - 7 \text{ MeV} \\ m_b = 4.1 - 4.4 \text{ GeV}, & m_s = 80 - 230 \text{ MeV}, & m_d = 4 - 12 \text{ MeV} \\ m_\tau = 1.777 \text{ GeV}, & m_\mu = 105.7 \text{ MeV}, & m_e = 0.511 \text{ MeV} \end{array} \tag{1.9}$$

and the quark mixing angles are known experimentally with a very good accuracy:[5]

$$\theta_{23}^q = (2.3 \pm 0.2)^\circ , \qquad \theta_{12}^q = (12.7 \pm 0.1)^\circ , \qquad \theta_{13}^q = (0.18 \pm 0.04)^\circ . \tag{1.10}$$

As for the neutrinos, information on their masses can be extracted directly from the ranges of δm^2_{atm} and δm^2_{sol} needed for the explanation of the AN and SN anomalies. Barring the less natural possibility that the neutrino mass eigenstates $\nu_{1,2,3}$ are strongly degenerate and assuming the mass hierarchy $m_3 > m_2 > m_1$, these data translate into the following values for neutrino masses:

$$m_3 = (4.7^{+3.0}_{-2.5}) \times 10^{-2} \text{ eV} , \qquad m_2 = (2.2^{+1.0}_{-0.5}) \times 10^{-3} \text{ eV} . \tag{1.11}$$

Hence, the neutrino mass hierarchy $m_3/m_2 \sim 10-50$ is similar to that of the charged leptons or down quarks. However, the magnitudes of the neutrino mixing angles[6]

$$\theta_{23}^l = (45 \pm 12.5)^\circ , \qquad \theta_{12}^l = (2.2 \pm 1.3)^\circ , \qquad \theta_{13}^l < (13 - 20)^\circ , \tag{1.12}$$

are in a dramatic contrast with the corresponding angles (1.10) in the quark mixing. In other words, the AN anomaly points to maximal 23 mixing in leptonic sector versus

[4] Following the tradition, for the heavy quarks t, b, c, we refer to their running masses respectively at $\mu = m_{t,b,c}$, and for the light quarks u, d, s – at $\mu = 1$ GeV. The quoted value of m_t corresponds to the 'pole' mass $M_t = 173.8 \pm 5.2$ GeV [8].

[5] We find it instructive to present the values of mixing angles in degrees, which correspond to the CKM matrix elements $|V_{us}| = 0.221 \pm 0.002$, $|V_{cb}| = 0.040 \pm 0.004$ and $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [8].

[6] As far as the V_{e3} element is concerned, for $\delta m^2_{\text{atm}} > 2 \times 10^{-3}$ eV^2 the limit $\theta_{13}^l < 13^\circ$ follows from the CHOOZ experiment [9]. But also for the δm^2_{atm} region not covered by CHOOZ, the recent fits disfavour θ_{13}^l larger than 20° or so [10]. In a whole, $\theta_{13}^l \approx 0$ provides the best data fit both for AN and SN cases [11].

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very small 23 mixing of quarks, and on the contrary, the MSW solution implies a very small 12 lepton mixing angle versus the reasonably large value of the Cabibbo angle.

As said above, the SM does not contain any theoretical input restricting the structure of the matrices $\mathbf{Y}_{u,d,e}$ and $\mathbf{Y}_\nu$ and thus fermion mass hierarchy and mixing pattern remain unexplained. Concerning the neutrinos, also the mass scale M remains a free parameter. One can only conclude that if the maximal constant in $\mathbf{Y}_\nu$ is order of the top Yukawa constant, $Y_3 \sim Y_t \sim 1$, then the mass value m_3 in (1.11) points to the scale $M \sim 10^{15}$ GeV, rather close to the grand unified scale. On the other hand, the drastic difference of the neutrino mixing pattern from that of the quarks at first glance suggests that the neutrino mass texture is very special and it indicates no similarity to that of the quarks and charged leptons. During the past years many models have been produced suggesting various exotic input textures for understanding the neutrino mixing pattern (e.g. ref. [12]; for a more generic discussions see refs. [11, 13]. There have been also attempts to obtain the desired pattern in the context of grand unification [14, 15, 16]).

In this paper we show that there is a *simple* and *coherent* way of understanding both the quark and neutrino mixing patterns within an unified framework. Our consideration is motivated by the following points. It is tempting to think that the intriguing empiric relations between the masses and mixing angles, such as the well-known formula for the Cabibbo angle $s^q_{12} = \sqrt{m_d/m_s}$, are not accidental and the fermion flavour structure is intrinsically connected to the peculiarities of some underlying theory which fully determines, or at least somehow constrains the form of the Yukawa matrices. The ideas of supersymmetry, grand unification and horizontal symmetry may constitute the essential ingredients of flavour physics and can be regarded as the present *Modus Operandi* for predictive model building (for a review on the fermion mass models see e.g. [17] and references therein).

In particular, relations between the fermion masses and mixing angles can be obtained by considering Yukawa matrix textures with reduced number of free parameters, putting certain elements to zero. For example, one can consider the popular texture suggested by Fritzsch [18], which implies that the fermion mass generation starts from the 3^{rd} family and proceeds to lighter families through the mixing terms:

$$\mathbf{Y}_{u,d,e} = \begin{pmatrix} 0 & A'_{u,d,e} & 0 \\ A_{u,d,e} & 0 & B'_{u,d,e} \\ 0 & B_{u,d,e} & C_{u,d,e} \end{pmatrix}, \tag{1.13}$$

where all elements are generically complex and obey the additional condition:

$$|A'_f| = |A_f|\,, \qquad |B'_f| = |B_f|\,; \qquad f = u, d, e\,. \tag{1.14}$$

This texture could emerge due to horizontal symmetry reasons, and the "symmetricity" property (1.14) can be motivated in the context of the left-right symmetric models [18], or of the $\mathrm{U}(3)_H$ horizontal symmetry [19] (see also the discussion in [20]).

This pattern has many striking properties. For example, it nicely links the observed value of the Cabibbo angle, $V_{us} \approx \sqrt{m_d/m_s}$, to the observed size of the CP-violation in the $K - \bar{K}$ system, moreover that the predicted magnitude $|V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}$ is also in good agreement with experiment. Unfortunately, the Fritzsch texture contains a strong conflict between the small value of V_{cb} and the large top mass and there is no parameter space in which these observables could be reconciled [21].

However, one can consider textures like (1.13) without the symmetricity in the 23 block of $\mathbf{Y}_d$, $B'_d \neq B_d$. Then one could achieve the small magnitude of V_{cb} at the price of taking the parameter $b_d = B'_d/B_d$ considerably larger than 1.[7] In the present paper we present a simple observation: in the context of the SU(5) grand unified theory such a choice naturally implies that the 23 mixing in the leptonic sector becomes large, and the parameter $b_e = B_e/B'_e$ increases in parallel with b_d since in SU(5) the Yukawa matrices are related as $\mathbf{Y}_e = \mathbf{Y}_d^T$, modulo certain Clebsch factors. Our point can be simply expressed as follows. If the down-quark and charged-lepton matrices had the symmetric Fritzsch texture, then we would have $\tan\theta_{23}^d = (m_s/m_b)^{1/2}$ and $\tan\theta_{23}^e = (m_\mu/m_\tau)^{1/2}$, which are both unsatisfactory: the former is too big as compared to $|V_{cb}|$, while the latter is too small for $|V_{\mu 3}|$. On the other hand, whenever the symmetricity condition is abandoned, none of these angles can be predicted in terms of mass ratios since their values now depend on the amount of asymmetry between the 23 and 32 entries, i.e. on the factors b_d and b_e. However, in the context of the SU(5) theory there emerges the following product rule:

$$\tan\theta_{23}^d \tan\theta_{23}^e \sim \left(\frac{m_\mu m_s}{m_\tau m_b}\right)^{1/2}. \qquad (1.15)$$

Therefore, if $\tan\theta_{23}^d$ becomes smaller than $(m_s/m_b)^{1/2}$, then $\tan\theta_{23}^e$ should correspondingly increase over $(m_\mu/m_\tau)^{1/2}$, and when the former reaches the value $|V_{cb}| \simeq 0.05$, the latter becomes ~ 1 (this happens for $b_{d,e} \sim 8$). Though these estimates are not precise (the exact expressions will be given in section 2), they qualitatively rather well demonstrate the "seesaw" like correspondence between the quark and lepton mixing angles whenewer their magnitudes are dominated by the rotation angles coming from the down fermions. A similar argument can be applied also to the 12 mixing:

$$\tan\theta_{12}^d \tan\theta_{12}^e \sim \left(\frac{m_e m_d}{m_\mu m_s}\right)^{1/2}, \qquad (1.16)$$

[7]A particular texture with the maximal asymmetry, $B'_d = C_d$, was suggested in ref. [22], and its relevance for small quark mixing angles was pointed out. The fact that in grand unified theories a similar asymmetry could lead to the large neutrino mixing was demonstrated in the $SO(10)$ based models [15].

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The relation $V_{us} \simeq (m_d/m_s)^{1/2}$ points that the 12 block of $\mathbf{Y}_d$ should be nearly symmetric, and hence we expect that $V_{e2} \sim (m_e/m_\mu)^{1/2}$. All of these will be discussed in details in the next section.

In principle, reasonable contributions to the mixing angles will emerge if also the upper quark and neutrino couplings, $\mathbf{Y}_u$ and $\mathbf{Y}_\nu$, have (symmetric) textures like (1.13). In this case the proper fit of the quark and lepton mixing patterns can be achieved for rather moderate asymmetry, $b_{d,e} \geq 2$. This possibility is discussed in section 3, where we also consider models with a horizontal symmetry which could provide the natural realization of the suggested texture. Finally, at the end we briefly outline our results and their implications.

2. Fritzsch-like $\mathbf{Y}_{d,e}$ and diagonal $\mathbf{Y}_{u,\nu}$ in SU(5)

In the SU(5) model the quark and lepton states of each family fit into the following multiplets: $\bar{5}_i = (d^c, l)_i$, $10_i = (u^c, e^c, q)_i$ (here and in the following the SU(5) indices are suppressed, and $i = 1, 2, 3$ is a family index). As for the Higgs doublets $\phi_{1,2}$, together with their colour triplet partners $T, \bar{T}$ they form the representations $H = (T, \phi_2) \sim 5$ and $\bar{H} = (\bar{T}, \phi_1) \sim \bar{5}$. The theory contains also the chiral superfield in the adjoint representation, $\Phi \sim 24$, which at the scale $M_G \sim 10^{16}$ GeV breaks the SU(5) symmetry down to SU(3) $\times$ SU(2) $\times$ U(1).

The Yukawa terms responsible for the fermion masses are the following:

$$\bar{H} 10_i \mathbf{G}^{ij} \bar{5}_j + H 10_i \mathbf{G}_u^{ij} 10_j + \frac{HH}{M} \bar{5}_i \mathbf{G}_\nu^{ij} \bar{5}_j \,, \tag{2.1}$$

where the Yukawa constant matrices $\mathbf{G}_u$ and $\mathbf{G}_\nu$ are symmetric due to SU(5) symmetry reasons while the form of $\mathbf{G}$ is not constrained. After the SU(5) symmetry breaking these terms reduce to the SM Yukawa couplings (1.3) with

$$\mathbf{Y}_e = \mathbf{G} \,, \qquad \mathbf{Y}_d = \mathbf{G}^T \,, \qquad \mathbf{Y}_u = \mathbf{G}_u \,, \qquad \mathbf{Y}_\nu = \mathbf{G}_\nu \,. \tag{2.2}$$

Without loss of generality, the matrices $\mathbf{G}_u$ and $\mathbf{G}_\nu$ can be taken diagonal, so that the weak mixing matrix in both the quark and leptonic sectors is determined by the unitary matrices rotating the left states of the down fermions: $V_q = U_d$ and $V_l = U_e^\dagger$. On the other hand, since $\mathbf{Y}_d = \mathbf{Y}_e^T$, we get that $U_d = U_e'$ and $U_e = U_d'$, so that the rotation angles of the left down quarks (charged leptons) are related to the unphysical angles rotating the right states of the charged leptons (down quarks).[8] Therefore, the smallness of the quark mixing angle θ_{23}^q does not necessarily imply the smallness of the leptonic mixing θ_{23}^l but rather the opposite, it can point to the large value of the latter.

[8]The latter have no physical significance for the low energy theory (the SM) and could be relevant only for the baryon number violating processes mediated by the SU(5) gauge fields or Higgses/Higgsinos.

Indeed, imagine that in the basis where $\mathbf{G}_u$ and $\mathbf{G}_\nu$ are diagonal:

$$\mathbf{G}_u = \mathbf{Y}_u^D = \begin{pmatrix} Y_u & 0 & 0 \\ 0 & Y_c & 0 \\ 0 & 0 & Y_t \end{pmatrix}, \qquad \mathbf{G}_\nu = \mathbf{Y}_\nu^D = \begin{pmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix}, \tag{2.3}$$

the matrix $\mathbf{G}$ has a Fritzsch-like form:

$$\mathbf{G} = \begin{pmatrix} 0 & G_{12} & 0 \\ G_{21} & 0 & G_{23} \\ 0 & G_{32} & G_{33} \end{pmatrix}. \tag{2.4}$$

In this case, as we have already remarked, the large value of the leptonic rotation angle is related to the smallness of the corresponding quark angle. However, if the entries of the matrix $\mathbf{G}$ are just constants, we would face the well-known problem of the down-quark and charged-lepton degeneracy in the SU(5) symmetry limit: since $\mathbf{Y}_d^D = \mathbf{Y}_e^D$, then we have $Y_{d,s,b} = Y_{e,\mu,\tau}$. Nevertheless, the $Y_b = Y_\tau$ unification is a definite success of the SUSY SU(5) GUT. After accounting for the renormalization running of the Yukawa constants from M_G to lower-energy scales, it provides a nice explanation of the magnitude of the bottom mass and its intimate relation to the large top mass. However, the other predictions $Y_s = Y_\mu$ and $Y_d = Y_e$ are wrong: they imply $m_s/m_d = m_\mu/m_e \simeq 200$, an order of magnitude in conflict with the current algebra estimate $m_s/m_d \simeq 20$.

It is natural, however, to consider that the $\mathbf{G}^{ij}$ are in fact operators dependent on the adjoint superfield of SU(5): $G_{ij} = G_{ij}(\Phi)$. The latter should be understood as expansion series $G_{ij}(\Phi) = G_{ij}^{(0)} + G_{ij}^{(1)} \frac{\Phi}{M_s} + \cdots$, where M_s is some fundamental scale larger than M_G (it can be e.g. the string scale or the scale where some GUT with larger symmetry reduces to SU(5)). In other words, one can assume that the couplings $\bar{H}10_i\mathbf{G}^{ij}\bar{5}_j$ contain the effective higher-order operators $\frac{\Phi}{M_s}\bar{H}10_i G_{ij}^{(1)}\bar{5}_j$ etc., just on the same footing as the last term in (2.1). Since in general the operator $\Phi \cdot H$ is represented by the tensor product $24 \times \bar{5} = \bar{5} + \overline{45}$, it can distinguish the corresponding entries in the matrices $\mathbf{Y}_e$ and $\mathbf{Y}_d$. Needless to say that in the field theory context such operators can be effectively induced from the renormalizable Lagrangian, by integrating out the additional superheavy states with masses $\sim M_s$ (so called Frogatt-Nielsen or *universal seesaw* mechanism [23]), much in the same way as the effective neutrino operator is obtained in the context of the seesaw mechanism by integrating out the heavy right-handed Majorana neutrinos. In addition, the ratio $\varepsilon = \langle\Phi\rangle/M_s$ can be used as a small parameter for understanding the fermion mass hierarchy.

Therefore, for the charged-lepton and the down-quark Yukawa matrices we get:

$$\mathbf{Y}_e = \begin{pmatrix} 0 & A'_e & 0 \\ A_e & 0 & B'_e \\ 0 & B_e & C_e \end{pmatrix}, \qquad \mathbf{Y}_d = \begin{pmatrix} 0 & A'_d & 0 \\ A_d & 0 & B'_d \\ 0 & B_d & C_d \end{pmatrix}, \tag{2.5}$$

With a proper redefinition of the fermion phases, all the elements in (2.5) can be made real and positive.

As we said above, generically the constants G_{ij} can be functions of the adjoint superfield Φ, so that the tensor product $G_{ij}(\Phi)\bar{H}$ can contain both $\bar{5}+\overline{45}$ channels and therefore the corresponding entries between $\mathbf{Y}_e$ and $\mathbf{Y}_d^T$ should be distinguished by some Clebsch coefficients of SU(5). However, for simplifying our analysis, and for designing more predictive ansatz, we impose some rather natural constraints. In particular, inspired by the success of $Y_b \simeq Y_\tau$ unification, we assume that G_{33} is a SU(5) singlet, so that $C_d = C_e = G_{33}$. Analogously, following another interesting relation $Y_d Y_s \simeq Y_e Y_\mu$ we assume that also G_{12} and G_{21} are SU(5) singlets. Furthermore, motivated by the celebrated formula for the Cabibbo angle $s^q_{12} \simeq \sqrt{m_d/m_s}$, we suppose that the matrix $\mathbf{G}$ is symmetric in the 12 block, $G_{12} = G_{21}$. All these imply that

$$\begin{aligned} &C_d = C_e \; (= C)\,, \\ &A_d = A'_d = A'_e = A_e \; (= A)\,. \end{aligned} \tag{2.6}$$

Thus only the $G_{23}(\Phi)$ and $G_{32}(\Phi)$ entries are left unconstrained. They contain non-trivial SU(5) Clebsches breaking the quark and lepton symmetry:

$$B'_d = k' B_e\,, \qquad B_d = k B'_e\,, \tag{2.7}$$

where the coefficients k and k' are not necessarily the same, since generically the tensor products $G_{23}(\Phi)\cdot\bar{H}$ and $G_{32}(\Phi)\cdot\bar{H}$ can emerge in different combinations of $\bar{5}$ and $\overline{45}$. In addition, these entries are not necessarily symmetric, and we introduce the asymmetry parameters $b_e = B_e/B'_e$ and $b_d = B'_d/B_d = \frac{k'}{k} b_e$. Then, identifying $B_e = B$ and $b_e = b$, the matrices $\mathbf{Y}_e$ and $\mathbf{Y}_d$ can be represented as:

$$\mathbf{Y}_e = \begin{pmatrix} 0 & A & 0 \\ A & 0 & \frac{1}{b}B \\ 0 & B & C \end{pmatrix}, \qquad \mathbf{Y}_d = \begin{pmatrix} 0 & A & 0 \\ A & 0 & k'B \\ 0 & \frac{k}{b}B & C \end{pmatrix}, \tag{2.8}$$

Therefore, we have an ansatz depending on six parameters, three Yukawa entries A, B, C and three Clebsch factors k, k' and b. As far as $\mathbf{Y}_u$ and $\mathbf{Y}_\nu$ are taken diagonal, the patterns of the quark and lepton mixings are completely determined by the form of $\mathbf{Y}_{d,e}$. The Yukawa eigenvalues $Y_{e,\mu,\tau}$ and $Y_{d,s,b}$ as well as the mixing angles $s^q_{12}, s^q_{23}, s^q_{13}$ and $s^l_{12}, s^l_{23}, s^l_{13}$ can be expressed in terms of the parameters in (2.8) and hence at the GUT scale there should emerge six relations between these physical quantities.

The GUT scale Yukawa constants are linked to the physical fermion masses (1.9) through the renormalization group equations (RGE). For moderate values of $\tan\beta = v_2/v_1$, one obtains at one-loop (see e.g. [24]):

$$\begin{aligned} m_u &= Y_u R_u \eta_u B_t^3 v_2\,, & m_d &= Y_d R_d \eta_d v_1\,, & m_e &= Y_e R_e v_1\,, \\ m_c &= Y_c R_u \eta_c B_t^3 v_2\,, & m_s &= Y_s R_d \eta_s v_1\,, & m_\mu &= Y_\mu R_e v_1\,, \\ m_t &= Y_t R_u B_t^6 v_2\,, & m_b &= Y_b R_d \eta_b B_t v_1\,, & m_\tau &= Y_\tau R_e v_1\,, \end{aligned} \tag{2.9}$$

where the factors $R_{u,d,e}$ account for the gauge-coupling induced running from the GUT scale $M_G \simeq 10^{16}$ GeV to the SUSY breaking scale $M_S \simeq M_t$, and the factors η_f encapsulate the QCD+QED running from M_S down to m_f for $f = b, c$ (or to $\mu = 1$ GeV for the light quarks $f = u, d, s$). Namely, for $\alpha_s(M_Z) = 0.118 \pm 0.005$ we have

$$\begin{aligned} R_u &= 3.33 \pm 0.07\,, & R_d &= 3.25 \pm 0.07\,, & R_e &= 1.49\,, \\ \eta_b &= 1.52 \pm 0.04\,, & \eta_c &= 2.02^{+0.16}_{-0.13}\,, & \eta_{u,d,s} &= 2.33^{+0.29}_{-0.21}\,. \end{aligned} \tag{2.10}$$

The factor B_t includes the running induced by the large top quark Yukawa constant ($Y_t \sim 1$):

$$B_t = \exp\left[-\frac{1}{16\pi^2}\int_{\ln M_S}^{\ln M_G} Y_t^2(\mu)\mathrm{d}(\ln\mu)\right]. \tag{2.11}$$

B_t as a function of the GUT scale value Y_t is shown in fig. 1. We see that for Y_t varying from a lower limit $Y_t = 0.5$ imposed by the top pole-mass value to a perturbativity limit $Y_t \approx 3$, the factor B_t decreases from 0.9 to 0.7. As for the CKM mixing angles, their physical values are related to their values at the GUT scale (labelled by superscript G) as follows:[9]

$$\begin{aligned} s_{12}^q &= s_{12}^{qG}\,, \\ s_{23}^q &= s_{23}^{qG} B_t^{-1}\,, \\ s_{13}^q &= s_{13}^{qG} B_t^{-1}\,. \end{aligned} \tag{2.12}$$

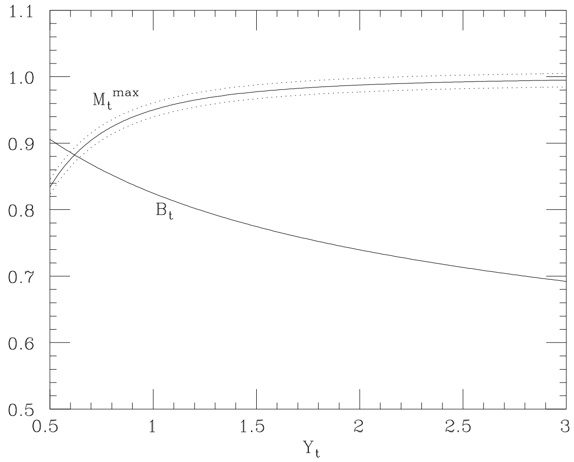


Figure 1: The decreasing solid line is the scaling factor B_t as a function of Y_t for $\alpha_s = 0.118$ (the sensitivity of B_t with respect to uncertainties in $\alpha_s(M_Z)$ is very low, around per cent or so). We also show the Y_t dependence of the top pole mass M_t^{max} (at $\sin\beta = 1$) for $\alpha_s = 0.118 \pm 0.005$.

Let us now discuss these predictions. Using the formulas given in the Appendix, we readily obtain the modified version of the $b - \tau$ Yukawa unification at the GUT scale:

$$C = Y_\tau\left[1 - (b_e + b_e^{-1})\frac{Y_\mu - Y_e}{Y_\tau}\right]^{1/2} = Y_b\left[1 - (b_d + b_d^{-1})\frac{Y_s - Y_d}{Y_b}\right]^{1/2} \tag{2.13}$$

and also the following relations:

$$A^2 C = Y_e Y_\mu Y_\tau = Y_d Y_s Y_b\,, \tag{2.14}$$

$$B_e B_e' = \frac{1}{b}B^2 = (Y_\mu - Y_e)Y_\tau\,, \qquad B_d B_d' = \frac{kk'}{b}B^2 = (Y_s - Y_d)Y_b\,. \tag{2.15}$$

In the following we directly substitute the Yukawa constant ratios with the corre-

[9] We shall not take into account the analogous RGEs [25] for the neutrino masses and mixing, since the experimental data (1.11) and (1.12) still contain big error bars and such an improvement is not justified. In addition, the RGE effects for the neutrino sector are model dependent: the renormalization of the constants $\mathbf{Y}_\nu$ depends on the concrete features of the underlying model effectively inducing the $d = 5$ operator (1.6).

sponding mass ratios when the latter are RGE invariant (cf. (2.9)), e.g. $Y_\mu/Y_\tau = m_\mu/m_\tau$, $Y_d/Y_s = m_d/m_s$, etc. Then, by dividing the squared eq. (2.13) on eq. (2.15), we obtain:

$$\frac{Y_b}{Y_s - Y_d} - (b_d + b_d^{-1}) = \frac{1}{kk'}\left[\frac{m_\tau}{m_\mu - m_e} - (b_e + b_e^{-1})\right] \tag{2.16}$$

to be rewritten as

$$\begin{aligned} \frac{Y_s - Y_d}{Y_b} &= \frac{kk'}{Z^2}\frac{m_\mu - m_e}{m_\tau} \approx 0.059\frac{kk'}{Z^2}\,; \\ Z &= \sqrt{1 - \left[(b_e + b_e^{-1}) - kk'(b_d + b_d^{-1})\right]\frac{m_\mu - m_e}{m_\tau}}\,. \end{aligned} \tag{2.17}$$

Substituting this expression back into eqs. (2.13) and (2.15) we get:

$$\frac{Y_b}{Y_\tau} = Z\,, \qquad \frac{Y_s - Y_d}{Y_\mu - Y_e} = \frac{kk'}{Z}\,, \tag{2.18}$$

Then by the RGE (2.9) we have for the physical masses:

$$\begin{aligned} m_b &= \frac{R_d \eta_b}{R_e} B_t Z m_\tau = B_t Z \cdot (5.90 \pm 0.30)\ \mathrm{GeV}\,, \\ m_s - m_d &= \frac{R_d \eta_d}{R_e}\frac{kk'}{Z}(m_\mu - m_e) = \frac{4kk'}{Z}\cdot(133 \pm 18)\ \mathrm{MeV}\,. \end{aligned} \tag{2.19}$$

with uncertainties related to the value of $\alpha_s(M_Z)$.

Analogously, from eqs. (2.14) we find:

$$\frac{m_s}{m_d} + \frac{m_d}{m_s} - 2 = \frac{(kk')^2}{Z}\left(\frac{m_\mu}{m_e} + \frac{m_e}{m_\mu} - 2\right) \approx 204.7\frac{(kk')^2}{Z}\,. \tag{2.20}$$

Therefore, for b_e, b_d varying within an order of magnitude, the value m_s/m_d can vary in the 'experimental' range $m_s/m_d = 17 - 25$, if kk' varies between $\frac{1}{5} - \frac{1}{3}$.

Let us turn now to the quark and lepton mixing. We parametrize the matrices U_d and U_e as in the appendix A: $U_e = O_{23}^e O_{13}^e O_{12}^e$ and $U_d = O_{23}^d O_{13}^d O_{12}^d$. Hence, for the angles $\theta_{23}^{e,d}$ which rotate the left-handed states we obtain:

$$\begin{aligned} \tan 2\theta_{23}^e &= 2\sqrt{b_e}\sqrt{\frac{m_\mu - m_e}{m_\tau}}\,\frac{\left[1 - (b_e + b_e^{-1})\frac{m_\mu - m_e}{m_\tau}\right]^{1/2}}{1 - 2b_e\frac{m_\mu - m_e}{m_\tau}}\,, \\ \tan 2\theta_{23}^d &= \frac{2}{\sqrt{b_d}}\sqrt{\frac{Y_s - Y_d}{Y_b}}\,\frac{\left[1 - (b_d + b_d^{-1})\frac{Y_s - Y_d}{Y_b}\right]^{1/2}}{1 - 2b_d^{-1}\frac{Y_s - Y_d}{Y_b}}\,, \end{aligned} \tag{2.21}$$

while the right rotation angles $\theta_{23}^{\prime e,d}$ are obtained from these expressions substituting respectively $b_e \to b_e^{-1}$ and $b_d \to b_d^{-1}$. For the other angles we have:

$$\begin{aligned} \tan 2\theta_{12}^e &= 2\sqrt{a_e}\sqrt{\frac{m_e}{m_\mu}}\,\frac{\left[1 - (a_e + a_e^{-1})\frac{m_e}{m_\mu}\right]^{1/2}}{1 - 2a_e\frac{m_e}{m_\mu}} \;\approx \frac{2}{\sqrt{c_{23}^e}}\sqrt{\frac{m_e}{m_\mu - m_e}} \approx \frac{0.140}{\sqrt{c_{23}^e}}\,, \\ \sin\theta_{13}^e &= \sqrt{a_e}\frac{s_{23}^{\prime e}}{c_{23}^{\prime e}}\frac{\sqrt{m_e m_\mu}}{m_\tau} = \left(\frac{m_e m_\mu^2}{b_e c_{23}^e m_\tau^3}\right)^{1/2} \le 10^{-3}\,, \end{aligned} \tag{2.22}$$

where $a_e = c'^e_{23}/c^e_{23}$, and

$$\tan 2\theta^d_{12} = \frac{2}{\sqrt{a_d}}\sqrt{\frac{m_d}{m_s}}\frac{\left[1-(a_d+a_d^{-1})\frac{m_d}{m_s}\right]^{1/2}}{1-2a_d^{-1}\frac{m_d}{m_s}} \approx 2\sqrt{c'^d_{23}}\sqrt{\frac{m_d}{m_s-m_d}}\,,$$
$$\frac{s^d_{13}}{s^d_{23}} = \frac{1}{\sqrt{a_d}}\frac{s'^d_{23}}{c'^d_{23}s^d_{23}}\sqrt{\frac{m_d}{m_s}\frac{Y_s}{Y_b}}\,, \qquad a_d = \frac{c^d_{23}}{c'^d_{23}}\,. \tag{2.23}$$

As far as $\mathbf{Y}_u$ is taken diagonal, the quark mixing is completely determined by the form of $\mathbf{Y}_d$, and hence $V_q = U_d$. Thus $U_d = O^d_{23}O^d_{13}O^d_{12}$ gives the CKM matrix V_q directly in the standard parametrization (1.5) with $\delta = \pi$. Therefore, we have $\theta^q_{ij} = \theta^d_{ij}$ and thus:

$$|V_{cb}| = c^d_{13}s^d_{23} \approx s^d_{23}\,, \qquad |V_{us}| = c^d_{13}s^d_{12} \approx s^d_{12}\,, \qquad |V_{ub}| = s^d_{13}\,. \tag{2.24}$$

As for the lepton mixing, the matrix $V_l = U_e^\dagger = O^{eT}_{12}O^{eT}_{13}O^{eT}_{23}$ appears in the parametrization transposed to (1.5). Hence, in standard parametrization the mixing angles read as:

$$|V_{\mu3}| = c^l_{13}s^l_{23} = |c^e_{12}s^e_{23} - s^e_{12}c^e_{23}s^e_{13}| \approx s^e_{23}\,,$$
$$|V_{e2}| = c^l_{13}s^l_{12} = |s^e_{12}c^e_{23} - c^e_{12}s^e_{23}s^e_{13}| \approx s^e_{12}c^e_{23}\,,$$
$$|V_{e3}| = s^l_{13} = |s^e_{12}s^e_{23} + c^e_{12}c^e_{23}s^e_{13}| \approx s^e_{12}s^e_{23}\,, \tag{2.25}$$

Hence, the lepton mixing angles are expressed in terms of their mass ratios and asymmetry parameter $b = b_e$.[10] In the fig. 2 we show the b-dependence of $|V_{\mu3}|$, $|V_{e2}|$ and $|V_{e3}|$. We also show the curves for the effective oscillation parameters $\sin^2 2\theta_{23} = 4|V_{\mu3}|^2(1-|V_{\mu3}|^2)$ and $\sin^2 2\theta_{12} = 4|V_{e2}|^2(1-|V_{e2}|^2)$. We see that for $b = 1$ the 23 mixing angle ($\theta^e_{23} = 13.5°$) is rather small for explaining the AN anomaly, while the 12 mixing ($\theta^e_{12} = 4.0°$) is somewhat above the upper limit obtained by the MSW fit of the SN data (cf. eq. (1.12)). However, for larger b, $|V_{\mu3}|$ increases roughly as $\sqrt{b}$ does and becomes maximal around $b = 8.4$, while $|V_{e2}|$ slowly decreases

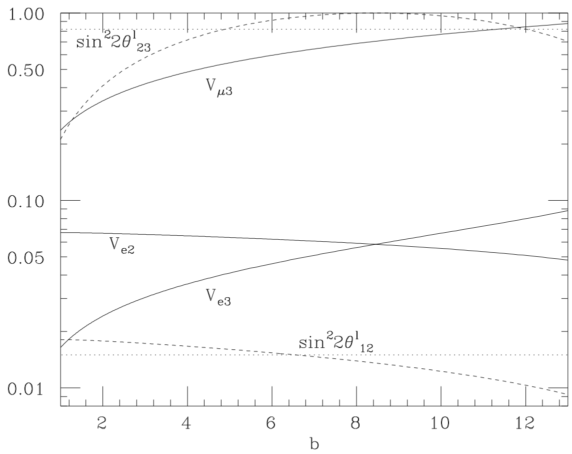


Figure 2: The lepton mixing angles V_{e2}, V_{e3} and $V_{\mu3}$ as functions of b (solid). The oscillation parameters $\sin^2 2\theta^l_{23}$ and $\sin^2 2\theta^l_{12}$ are also shown (dash), and the experimental limits $\sin^2 2\theta^l_{23} > 0.82$ and $\sin^2 2\theta^l_{12} < 1.5 \cdot 10^{-2}$ are delimited by the dotted lines.

[10] The 'critical' value of b when $B_e = C$ is given by the equation $2b + b^{-1} = m_\tau(m_\mu - m_e)^{-1}$, and so $b = 8.4$.

(roughly as $\sqrt{c_{23}^e}$). Thus, the AN bound, $\sin^2 2\theta_{23} > 0.82$, requires $5 < b < 12$, while the MSW fit for SN data is recovered at $b > 7$, when $\sin^2 2\theta_{12}$ drops below $1.5 \cdot 10^{-2}$.[11] Therefore, the relevant values of b are somewhere between 7 and 12, in which range $\sin^2 2\theta_{12}$ varies from $1.5 \cdot 10^{-2}$ to $1.0 \cdot 10^{-2}$, and $\sin\theta_{13}^l$ varies from 0.05 to 0.08, well below the upper bound of eq. (1.12). For example, for the case $B_e = C$ we have $b = 8.4$, and nearly maximal 23 mixing: $V_{\mu 3} \approx 0.7$, while $V_{e2}, V_{e3} \approx 0.055$.

Therefore, we conclude that in the basis where the neutrino masses are diagonal, the pattern of the lepton mixing required by the AN and SN anomalies can be perfectly described if the charged lepton mass matrix has a Fritzsch-like form (2.8) with an asymmetry parameter $b = 7 - 12$. We also see that in this range θ_{13}^l remains rather small, between $(3-5)^\circ$. This range, however, in the case of δm_{23}^2 close to the upper bound in (1.1), can be of interest for the experimental search of $\nu_e \to \nu_\tau$ oscillation in the future CERN Neutrino Factory [27].

As far as the quarks are concerned, from eqs. (2.19)–(2.20) and (2.22)–(2.23) we see that their masses and mixing angles are all expressed in terms of the lepton mass ratios and three Clebsch factors k, k' and b. In addition, the physical mass of bottom depends, through the factor B_t, on the top Yukawa constant Y_t at the GUT scale. Notice also that the expression (2.21) defines V_{cb} at the GUT scale, and for obtaining its physical value at lower scales one has to take into account the factor B_t^{-1}, while V_{us} and V_{ub}/V_{cb} are RGE invariant.

One can see that for $k \sim k'$ and large values of b, ($b = 7 - 12$ as required from the lepton mixing) our ansatz can give a satisfactory explanation also to the down quark mass and mixing pattern. In particular, in fig. 3a we show the dependence of the quark mixing angles on the parameter $b = b_e$ in the case of $b_d = b_e$, i.e. $k = k'$.[12] The complementary fig. 3b exhibites the b-dependence of the s-quark mass and m_s/m_d ratio. The same figure contains information on the value Y_t needed for the experimental value of the bottom mass. Since for $k = k'$, we have $Z = [1 - 0.059(b + b^{-1})(1 - k^2)]^{1/2}$, this provides a substantial correction to the $b - \tau$ Yukawa

[11] In fact, the value $\sin^2 2\theta_{\rm sol} = 1.5 \times 10^{-2}$ can be considered as acceptable in the context of the solar models with the boron neutrino flux exceeding its 'standard solar model' value by a factor 2 or so [26].

[12] In terms of the Φ-dependent higher order operators this means that the tensor product $G_{23}(\Phi) \cdot \bar{H}$ and $G_{32}(\Phi) \cdot \bar{H}$ project the same mixture of the 5 and 45 Clebsches on 23 and 32 entries. Furthermore, the value $k = k' = 1/2$ for the Clebsch factors can emerge if the effective operators $\frac{\Phi}{M}\bar{5}\bar{H}10$ is induced by the Frogatt-Nielsen mechanism [23] involving the heavy states in $10 + \overline{10}$ representation, and Φ has the VEV of the form $\propto \mathrm{Diag}(0,0,0,1,1)$ [28]. This means that Φ is taken as a reducible $1 + 24$ representation instead of the pure adjoint. Such a situation should not come up as a surprise if the SU(5) theory is obtained from a theory with higher-symmetry group, say from $SO(10)$ or $\mathrm{SU}(5) \times \mathrm{SU}(5)$. Then the relevant combination contained in the representations 45 (for $SO(10)$) or $(5,5)$ (for $\mathrm{SU}(5) \times \mathrm{SU}(5)$) is indeed $1 + 24$. It is worth remarking also, that another similar field $\Sigma \sim 1 + 24$ with a complementary missing VEV structure $\langle\Sigma\rangle \propto (1,1,1,0,0)$ can be used for the solution of the doublet-triplet splitting problem.

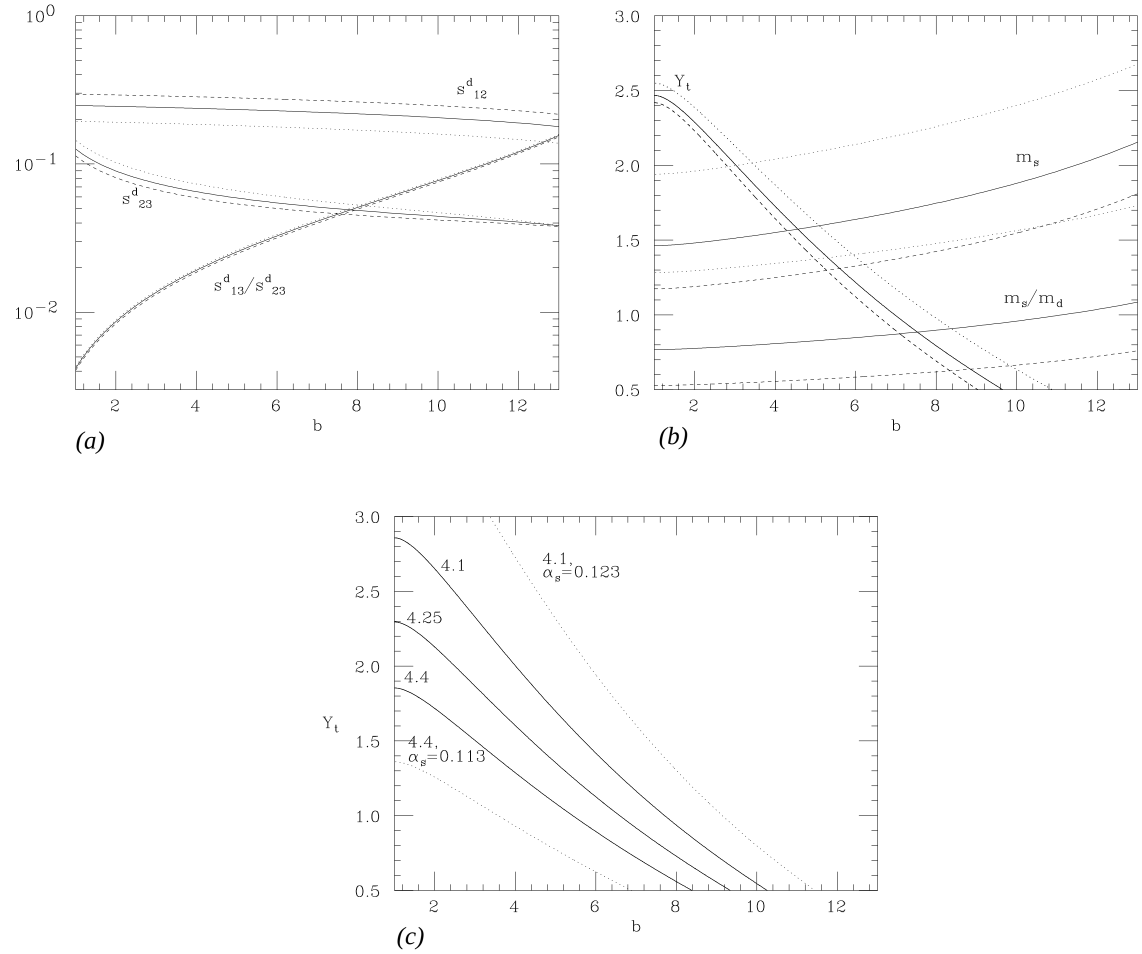
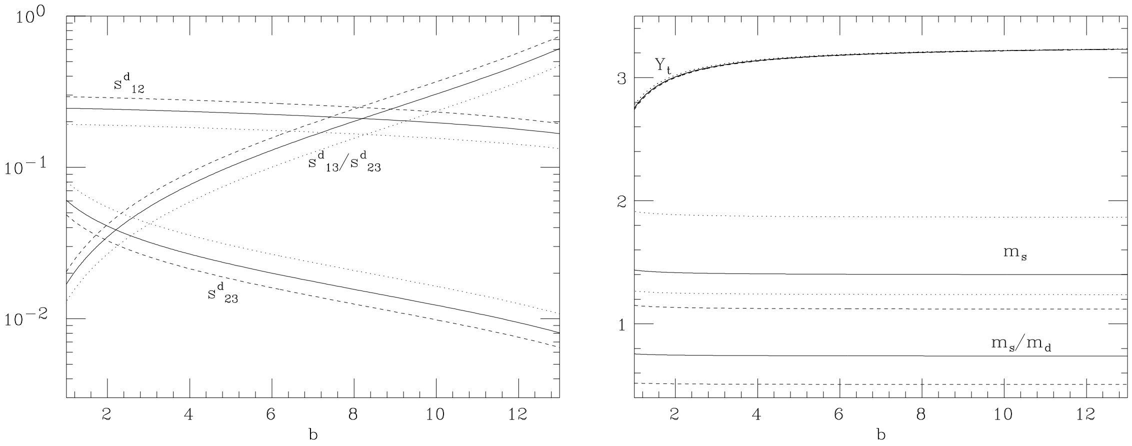


Figure 3: The quark mixing angles (fig 3a) and the down-quark masses (fig. 3b) as functions of $b = b_e$ in the case $b_d = b$ (i.e. $k' = k$). Three different values are considered: $k^2 = 1/4$ (solid), $k^2 = 1/3$ (dott) and $k^2 = 1/5$ (dash). All mixing angles shown in Fig. 3a are evaluated at the GUT scale. In fig. 3b. the strange mass m_s(1 GeV) is shown in units of 100 MeV and the ratio m_s/m_d in units of 20. In fig. 3b we also show iso-contours for $m_b = 4.25$ GeV in the $b - Y_t$ plane (for $\alpha_s = 0.118$). In fact, these curves signify the implicit dependence of Y_t on b in the context of the ansatz. The corresponding values of B_t^{-1} needed to rescale s^d_{23} are to be taken from the fig. 1. The effect of the uncertainties in α_s and m_b for Y_t is presented in fig. 3c (for $k = k' = 1/2$). The solid iso-contours correspond to $m_b = 4.25 \pm 0.15$ for $\alpha_s = 0.118$, and marginal cases $m_b = 4.1$, $\alpha_s = 0.123$, and $m_b = 4.4$, $\alpha_s = 0.113$ are outlined by the dotted contours.

unification for large b. Therefore, in this situation rather small values $Y_t \sim 0.5 - 1$ are needed to obtain the correct physical mass of the bottom.

We observe that indeed $k = 1/2$ provides the most acceptable description for all these data, in comparison to the cases $k^2 = 1/3$ or $1/5$ which are also shown. Thus, a global inspection of figs. 2 and 3 indicates that this case with $b = 7 - 12$ offers us a quite realistic picture for the quark and neutrino masses and mixing.



(a) *(b)*

Figure 4: The same as in the fig. $3a$ and fig. $3b$, for $k' = 1$ and $k = 1/4$ (solid) $k = 1/3$ (dott) and $k = 1/5$ (dash). Also here $b = b_e$ (and $b_d = \frac{1}{k}b$). In this case the factor Z is close to 1 and thus $Y_b = Y_\tau$, which requires larger values of Y_t, at the margin of the perturbative regime. This is in drastic contrast to what is exhibited in fig. $3b$ (case $k = k'$).

In fig. 4 we show the same for different choice of the Clebsches: $k' = 1$ ($b_d = k^{-1}b$) and $k = 1/3, 1/4$ and $1/5$. We see that the proper picture of the quark masses and mixing now requires rather small values of b, $b = 3-4$, which is incompatible with the $b = 7-12$ as it is dictated by the neutrino mixing pattern. One can also show that the case with $k = 1$ ($b_d = k'b$) leads to even less realistic picture.

The following remark is in order. Much on the same footing, one could consider also the ansatz when the matrices $\mathbf{G}_u$ and $\mathbf{G}_\nu$ are diagonal, $\mathbf{G}_u = \mathbf{Y}_u^D$ and $\mathbf{G}_\nu = \mathbf{Y}_\nu^D$, and the matrix $\mathbf{G}$ has the form

$$\mathbf{G} = \rho\mathbf{Y}_u^D + \mathbf{A}(\Phi)\,, \qquad \mathbf{A} = \begin{pmatrix} 0 & A_{12} & 0 \\ A_{21} & 0 & A_{23} \\ 0 & A_{32} & 0 \end{pmatrix}, \tag{2.26}$$

with $\mathbf{A}$ being a matrix with the Φ-field dependent off-diagonal (complex) entries.[13] This pattern is reminescent of the once popular texture proposed by Stech, and independently by Chkareuli and one of the authors [30]. However, in these papers the matrix $\mathbf{A}$ has been taken antisymmetric, which choice was soon excluded by the experimental data. Committing a minimal modification of the original version of the ansatz [30], one could assume that the 12 block of $\mathbf{A}$ remains antisymmetric, $A_{12} = -A_{21}$, and Φ independent, whereas Φ-dependent entries $A_{23} \neq A_{32}$ are strongly

[13] Interestingly, within the same philosophy of taking the relevant terms diagonal between the similar multiplets ($\bar{5}$'s and 10's), one could allow also the R-violating couplings $\lambda_{ik}(\Phi)\bar{5}_i\bar{5}_i 10_k$ which are diagonal between 5-plets for any k. In this case, the R-violating term $\lambda_{iik} l_i d^c_i q_k$ would emerge in the low energy theory which could also contribute the neutrino masses, while the other two terms lle^c and $d^c d^c u^c$ would vanish by symmetry reasons [29].

asymmetric. The GUT models leading to this ansatz and its complete analysis will be presented elsewhere. It is easy to see that the patterns of the quark and lepton mixing essentially remain the same as in the Fritzsch-like ansatz for $\mathbf{G}$ considered above – the presence of the diagonal 22 and 11 entries in $\mathbf{G}$ with the hierarchy stepped as $Y_u : Y_c : Y_t$ will lead only to small corrections. However, the CP-violating phase in this case would not vanish.

3. Give Fritzsch texture a chance also for $\mathbf{Y}_u$ and $\mathbf{Y}_\nu$

In the above we have assumed that the matrix $\mathbf{G}$ has a Fritzsch-like texture while $\mathbf{G}^u$ and $\mathbf{G}^\nu$ are diagonal. However, in the context of models in which the form of $\mathbf{G}$ is fixed by some underlying horizontal symmetry reasons, it would seem more natural that $\mathbf{G}^u$ and $\mathbf{G}^\nu$ also have a similar form:

$$\mathbf{G}_u = \begin{pmatrix} 0 & G^u_{12} & 0 \\ G^u_{21} & 0 & G^u_{23} \\ 0 & G^u_{32} & G^u_{33} \end{pmatrix}, \qquad \mathbf{G}_\nu = \begin{pmatrix} 0 & G^\nu_{12} & 0 \\ G^\nu_{12} & 0 & G^\nu_{23} \\ 0 & G^\nu_{23} & G^\nu_{33} \end{pmatrix}, \tag{3.1}$$

Clearly, if the entries in $\mathbf{G}_u$ were SU(5) singlets, this matrix should be symmetric. More generally, for Φ dependent entries, this is not true anymore, since the symmetric contribution can be induced by terms containing the tensor product $\Phi \cdot H$ in the 5-channel and the antisymmetric ones by terms containing $\Phi \cdot H$ in the 45-channel. As for $\mathbf{G}_\nu$, clearly only the symmetric terms are relevant for the neutrino mass matrices. For simplicity, in the next we assume that also $\mathbf{G}_u$ is symmetric.

Such a scenario would provide some different features. In the model with diagonal $\mathbf{G}^u$ and $\mathbf{G}^\nu$ we have been forced to take too big an asymmetry between the 23 and 32 entries of $\mathbf{G}$, $b_{e,d} \lesssim 10$. In the case in which also $\mathbf{G}^u$ and $\mathbf{G}^\nu$ have the form (3.1), smaller values of $b_{e,d}$ can suffice since now the mixing angles will be contributed also by the unitary matrices U_u and U_ν: $V_q = U_u^\dagger U_d$ and $V_l = U_e^\dagger U_\nu$. Therefore, for the CKM mixing angles we have:

$$|V_{cb}| = s^q_{23} \approx \left| s^d_{23} - e^{i\varphi} s^u_{23} \right|, \qquad |V_{us}| = s^q_{12} \approx \left| s^d_{12} - e^{i\delta} s^u_{12} \right|, \qquad \left| \frac{V_{ub}}{V_{cb}} \right| \approx s^u_{12}, \tag{3.2}$$

where the phases φ, δ etc are combinations of the independent phases in the Yukawa matrices. The expressions for θ^d_{23} and θ^d_{12} are the same as in (2.21) and (2.23), while θ^u_{23}, θ^u_{12} are the analogous angles diagonalizing $\mathbf{Y}_u$: $\tan\theta^u_{23} = \sqrt{Y_c/Y_t}$ and $\tan\theta^u_{12} = \sqrt{m_u/m_c}$. The dependence of θ^d_{23} and θ^d_{12} on the parameter b for the cases $k = k'$ and $k' = 1$ can be read out respectively from figs. $3a$ and $4a$, and for small values b the contribution of s^d_{13} in V_{ub}/V_{cb} can be neglected.

Therefore, by varying the phase φ from 0 to π, the value of the 23 mixing angle in the CKM matrix can vary between its minimal and maximal possible values:

$$\theta^{q(\mp)}_{23} = \theta^d_{23} \mp \theta^u_{23}\,. \tag{3.3}$$

Analogously, for the leptonic mixing we have

$$\theta_{23}^{l(\mp)} = \theta_{23}^{e} \mp \theta_{23}^{\nu}\,, \qquad (3.4)$$

where $\tan\theta_{23}^{\nu} = \sqrt{m_2/m_3}$. Thus, for the range of the neutrino masses indicated in (1.11) we obtain $\theta_{23}^{\nu} = (12.2^{+8.6}_{-3.7})^{\circ}$. In case of moderate asymmetry in $\mathbf{Y}_{d,e}$, the entries in (3.3) are big as compared to the experimental value of θ_{23}^{q} while each of the entries in (3.4) is too small for the magnitude of θ_{23}^{l} required by the AN oscillation. However, by properly tuning the phases, θ_{23}^{q} can get close to $\theta_{23}^{q(-)} = \theta_{23}^{d} - \theta_{23}^{u}$ while θ_{23}^{l} can approach $\theta_{23}^{l(+)} = \theta_{23}^{e} + \theta_{23}^{\nu}$, In other words, the angles in the quark and lepton sectors could have negative and positive interference, respectively. Therefore, even for smaller values of $b_{e,d}$, one could achieve a proper fit of the mixing angles.

Nevertheless, it is well known that the case $b_{e,d} = 1$, corresponding to the original Fritzsch texture, is fully excluded. In this case we have $s_{23}^{d} \approx \sqrt{Y_s/Y_b}$ which implies a sharp conflict between the values of $|V_{cb}|$ and the large top mass M_t. Namely, the fig. 5 shows that for the most conservative bound $M_t > 160$ GeV, even the least possible value $\theta_{23}^{q(-)} = \theta_{23}^{d} - \theta_{23}^{u}$ exceeds its experimental range by a factor 2 or so (see the dotted curves).

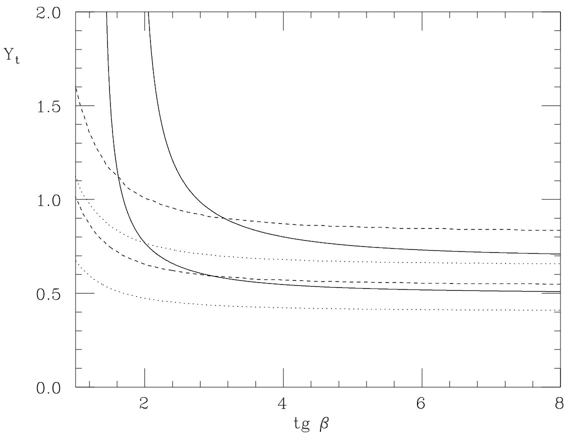


Figure 5: The solid isocontours for the top pole-mass correspond to $M_t = 174$ GeV (upper curve) and $M_t = 160$ GeV (lower curve). The dotted iso-contours correspond to $s_{23}^{q(-)} = 0.08$ (upper) and $s_{23}^{q(-)} = 0.07$ (lower) for the case $b_d = 1$ (Fritzsch texture). This plot demonstrates that even for the marginal values of quark masses used as input ($m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $m_s = 100$ MeV), the Fritzsch ansatz is completely excluded: the magnitude of $|V_{cb}|$ exceeds its experimental value at least by a factor of 2. On the contrary, the case of $b_d = 2$ (asymmetric Fritzsch texture [17]) can be accomodated: the dashed curves are isocontours for $s_{23}^{q(-)} = 0.044$ (upper) and $s_{23}^{q(-)} = 0.036$ (lower) and they are perfectly compatible with the experimental range of M_t. Let us remark, however, that $s_{23}^{q(-)}$ represents the least possible value of $|V_{cb}|$ (from the destructive interference between s_{23}^{d} and s_{23}^{u}), hence for arbitrary phases φ the typical values of $|V_{cb}|$ are larger. Clearly, for larger values of b_d the allowed range for φ becomes also larger.

Neither in the leptonic sector it is possible to fully reproduce the desired pattern (1.12). For $b_e = 1$ we have $\tan\theta_{23}^{e} = \sqrt{m_\mu/m_\tau}$, i.e. $\theta_{23}^{e} = 13.6^{\circ}$. Therefore, only the maximal value $\theta_{23}^{l(+)} = \theta_{23}^{e} + \theta_{23}^{\nu}$ can marginally satisfy the lower bound of (1.12).[14] On the other hand, now the 12 mixing is contributed also by the angle $\tan\theta_{12}^{\nu} = \sqrt{m_1/m_2}$ and therefore the unknown value of m_1 makes invalid the expression for s_{12}^{l}.

[14]This possibility was remarked in ref. [31].

However, by taking the matrices $\mathbf{Y}_{e,d}$ somewhat asymmetric in the 23 block, the situation can improve significantly. Already the choice $b_{e,d} = 2$ can suffice (such a possibility was suggested e.g. in ref. [17], where this factor 2 was obtained as a horizontal $\mathrm{U}(3)_H$ symmetry breaking Clebsch). In this case we have $s_{23}^d \approx \sqrt{(Y_s/2Y_b)}$ and then the value of $\theta_{23}^{q(-)} = \theta_{23}^d - \theta_{23}^u$ can perfectly agree with the large M_t, as it is demonstrated in the fig. 5 (see the dashed curves). On the other hand, now we obtain also $s_{23}^e \approx \sqrt{(2m_\mu/m_\tau)}$, i.e. $\theta_{23}^e = 23^\circ$, and thus the value of $\theta_{23}^{l(+)} = \theta_{23}^e + \theta_{23}^\nu$ can fit well into the range (1.12) required by AN oscillation.

In the above we have assumed that $k = k'$, where k and k' are the SU(5) breaking Clebsches defined as in eq. (2.7). However, in general these Clebsch factors have no reason to be the same. Nevertheless, for the dominant contributions θ_{23}^d and θ_{23}^e to the 23 quark and leptonic mixing angles (3.3) and (3.4) it emerges the "seesaw" like product rule:

$$\tan\theta_{23}^d \tan\theta_{23}^e \simeq \left(\frac{k}{k'}\right)^{1/2} \left(\frac{m_\mu m_s}{m_\tau m_b}\right)^{1/2}. \tag{3.5}$$

Barring unusual conspiracies, one can expect both k and k' to be ≤ 1 and then, bearing also in mind that $kk' \simeq (Y_s/Y_\mu) \sim 1/3 - 1/4$, the factor $(k/k')^{1/2}$ appears to be ~ 1. Even in rather extreme case $k = 1/3$ and $k' = 1$, this factor is about 0.6.

At this point some natural questions emerge, concerning the reasons for fermion Yukawa constants to have such Fritzsch-like textures, and the motivations underlying our assumptions. However, it is already known in the literature that such textures can naturally occur due to a horizontal symmetry (e.g. [19, 17]). Some very predictive schemes, based on the concept of the horizontal $\mathrm{U}(3)_H$ symmetry [32], and fully demonstrating the features discussed here, will be presented elsewhere. Here we confine ourselves to the discussion of theories with a simpler field content based on the horizontal $\mathrm{U}(2)_H$ symmetry [33, 34].[15]

Indeed, consider a theory based on the $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ symmetry where $\mathrm{U}(2)_H = \mathrm{SU}(2)_H \times \mathrm{U}(1)_H$ stands for a horizontal symmetry unifying the first two families in a doublet: $\bar{5}_\alpha = (\bar{5}, 2)$, $10_\alpha = (10, 2)$, $\alpha = 1, 2$, with an $\mathrm{U}(1)_H$ hypercharge $H = -1$, while the third family $(\bar{5}_3, 10_3)$ is a U(2) singlet with $H = 0$ [34]. The Higgs 5-plets are singlet of $\mathrm{U}(2)_H$, $H \sim (5, 1)$ and $\bar{H} \sim (\bar{5}, 1)$, and the theory contains also some amount of SU(5) singlets $S \sim (1, 1)$ and adjoints $\Phi \sim (24, 1)$, with VEVs of the order of $M \sim M_G \simeq 10^{16}$ GeV. For the horizontal symmetry breaking one can introduce the simplest set of the Higgs superfields: a doublet $\phi^\alpha = (1, 2)$ with $H = 1$ and a singlet $A = (1, 1)$ with $H = 2$, having nonzero VEVs $\langle\phi^\alpha\rangle = V_\phi \delta_2^\alpha$ and $\langle A\rangle = V_A$, smaller than M_G.

The third generation can get masses through the Yukawa couplings $H 10_3 10_3 + \bar{H} 10_3 \bar{5}_3$ while the masses of the first two generations emerge from the effective op-

[15] Applications of the U(2) horizontal symmetry for the neutrino mass and mixing pattern were discussed also in refs. [16], but in a different context.

erators induced by the Frogatt-Nielsen mechanism after integrating out some heavy vector-like states in representations $10+\overline{10}$ and $\bar{5}+5$ [23]. Following ref. [34], the latter can be chosen as the $\mathrm{U}(2)_H$ doublets $T^\alpha+\bar{T}_\alpha$ (ten-plets) and $\bar{F}^\alpha+F_\alpha$ (Five-plets). For the neutrino masses, we add also the SU(5) singlet fermions (right-handed neutrinos) $N^\alpha+\bar{N}_\alpha$, $a=1,2$, and $N+\bar{N}$. All these get large ($M\sim M_G$) masses from couplings to the fields S and Φ:

$$(S+\Phi)T^\alpha\bar{T}_\alpha+(S+\Phi)\bar{F}^\alpha F_\alpha+S(N^\alpha\bar{N}_\alpha+N^2+N\bar{N}+\bar{N}^2)\,. \tag{3.6}$$

Therefore, the heavy masses in the ten- and five-plets exhibite the SU(5) breaking between various $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ fragments with different U(1) hypercherges Y – they are respectively $M_T(1+x_TY)$ and $M_F(1+x_FY)$. Then the mass terms of the first two generations emerge by the *seesaw* mixing with the heavy ones:

$$\begin{aligned}&H10_\alpha T^\alpha+\bar{H}\bar{5}_\alpha T^\alpha+\bar{H}10_\alpha\bar{F}^\alpha+H\bar{5}_\alpha N^\alpha+H\bar{5}_3N\,,\\&A\varepsilon^{\alpha\beta}(\bar{T}_\alpha10_\beta+F_\alpha\bar{5}_\beta)+\phi^\alpha(\bar{T}_\alpha10_3+F_\alpha\bar{5}_3)+\phi^\alpha\bar{N}_\alpha(N+\bar{N})\,.\end{aligned} \tag{3.7}$$

For example, the effective operator for the neutrino masses appears in a form:

$$\frac{HH}{S}\bar{5}_3\bar{5}_3+\frac{HH\phi^\alpha}{S^2}\bar{5}_\alpha\bar{5}_3\,. \tag{3.8}$$

Therefore, the mass of the heaviest neutrino ν_3 could naturally emerge in the needed range around $m_3\sim4\cdot10^{-2}$ eV (cf. (1.11)). In fact, literally in the context of the effective operator (3.8) this implies $\langle S\rangle\sim10^{15}$ GeV rather than the GUT scale $M_G\simeq10^{16}$ GeV, but in terms of the renormalizable couplings (3.6) this mismatch can be easily understood e.g. due to some difference of the relevant coupling constants.[16]

Hence, after integrating out the heavy states, we arrive at the following Fritzsch-like textures for the the Yukawa couplings:

$$\mathbf{Y}_d=\begin{pmatrix}0&-A_d&0\\A_d&0&B'_d\\0&B_d&C\end{pmatrix},\qquad\mathbf{Y}_u=\begin{pmatrix}0&-A_u&0\\A_u&0&B'_u\\0&B_u&C_u\end{pmatrix}, \tag{3.9}$$

and

$$\mathbf{Y}_e=\begin{pmatrix}0&A_e&0\\-A_e&0&B'_e\\0&B_e&C\end{pmatrix},\qquad\mathbf{Y}_\nu=\begin{pmatrix}0&0&0\\0&0&B_\nu\\0&B_\nu&C_\nu\end{pmatrix}, \tag{3.10}$$

with 33 elements C_u and C_ν naturally being ~1, while C should be somewhat smaller

[16]Notice, that all couplings contained in the theory are trilinear, and thus they are invariant under the continuous R-symmetry with all superfields having R-charge 1 and the superpotential having the R-charge 3. From one side, this forbides the direct mass terms of superfields T, F and N to be of the order of the Planck scale. On the other hand, the superpotential terms of the GUT superfields S and Φ should contain only the trilinear terms $S^3+S\Phi^2+\Phi^3$. Then the mass parameter giving rise to their VEVs could emerge from the coupling $SQ\bar{Q}$ to the fermions Q, $\bar{Q}$ of the strongly coupled gauge sector, through the linear term Λ^2S emerging due to the dynamical condensation $\langle\bar{Q}Q\rangle=\Lambda^2$.

unless $\tan\beta$ is very large. Interestingly, when $r = M_T/M_F \ll 1$ and $x_T \ll 1$, the other Yukawa entries have the following hierarchy:

$$\begin{aligned}
&A_d \approx A_e = A \sim \frac{V_A}{M_T}\,, && A_u \sim x_T A\,, \\
&B'_u \approx B_u \sim \frac{V_\phi}{M_T}\,, && B'_d \approx B_e \sim \frac{V_\phi}{M_T}\,, \\
&B_\nu \sim V_\phi/M_N\,, && B_d, B'_e \sim \frac{V_\phi}{M_F} \sim r B_e\,.
\end{aligned} \tag{3.11}$$

Thus, we have $k' \approx 1$ and $b \sim 1/r$, since the asymmetry in the 23 block of $\mathbf{Y}_{e,d}$ is due to the mass difference between heavy Ten- and Five-plets. In addition, if $x_F \sim 1$, then $k = B_d/B'_e \neq 1$ and in particular it could be chosen around $1/3 - 1/4$. The implications of such ansatz for the quark masses and mixing was discussed in great details in ref. [34].

We would like to add the following remark. As was already said, now one can naturally achieve the large 23 mixing in the lepton sector even in the case of smaller asymmetry in $\mathbf{Y}_e$, already starting from $b \sim 1$, since now besides the (b-dependent) charged lepton angle θ^e_{23} it is contributed also by the neutrino angle θ^ν_{23} from the matrix $\mathbf{Y}_\nu$ ($\tan\theta^\nu_{23} = \sqrt{m_2/m_3}$). In addition, in this model we have $A_\nu = 0$, since there can be no antisymmetric entry for the neutrino Majorana masses. Hence, the eigenstate ν_1 remains massless, $m_1 = 0$, and therefore the 12 mixing angle in leptonic sector can be predicted.

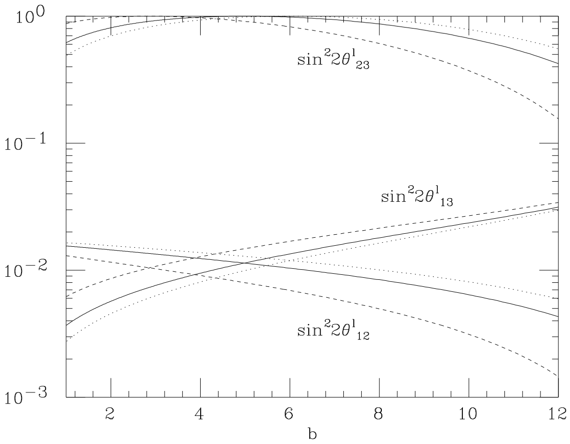


Figure 6: The lepton mixing angles as functions of b for positive interference between θ^e_{23} (depending on b as in eq. (2.21)) and $\theta^\nu_{23} = \arctan(\sqrt{m_2/m_3})$. The solid line corresponds $\theta^\nu_{23} = 12.2^\circ$ (central values of $m_{2,3}$ in (1.11)), while the marginal possible values 20.8° (dash) and 8.5° (dott) are also shown which account for the uncertainties in (1.11). This plot shows how small the parameter $\sin^2 2\theta^l_{12}$ could become for relevant values of $\sin^2 2\theta^l_{23}$.

In particular, now we have $|V_{e2}| = s^l_{12} \approx s^e_{12} c^l_{23}$, and assuming the positive interference in eq. (3.4), we find that θ^l_{23} falls in the range needed for AN oscillation, $\theta^l_{23} = (33-57)^\circ$, already for moderate values of b for which $c^e_{23} \approx 1$ and so $s^e_{12} \approx \sqrt{m_e/m_\mu}$. This implies that for the angle θ^l_{23} in the range needed for the AN oscillation (i.e. $c^l_{23} = 0.83 - 0.55$), the angle θ^l_{12} is entirely contained in the range required by SN: $\sin^2 2\theta^l_{12} \approx 4(m_e/m_\mu)(c^l_{23})^2 = (1.4 - 0.6)\times 10^{-2}$, and moreover it can approach also the best MSW fit value of $\sin^2 2\theta_{\rm sol}$ [4]. The b-dependence of the lepton mixing pattern for the interval $\theta^\nu_{23} = (12.2^{+8.6}_{-3.7})^\circ$ which accounts for the uncertainties in the neutrino masses (1.11) is shown in fig. 6.

4. Discussion and outlook

We have argued that the neutrino mixing pattern required by the solutions of the AN and SN anomalies can be obtained in a rather natural way in the context of the SU(5) grand unification, assuming that contributions to the leptonic mixing angles emerge completely or domininantly from the charged-lepton mass matrices which have Fritzsch-like textures with strongly asymmetric 23 block and nearly symmetric 12 block. In this case, as far as $\mathbf{Y}_d \sim \mathbf{Y}_e^T$ modulo SU(5) symmetry breaking Clebsch factors, the large value of the 23 leptonic mixing angle can be nicely linked to the small value of the 23 mixing angle in the quark sector. Namely, for the dominant contributions to the quark and lepton angles of 23 mixing it emerges the "seesaw" balance rule (1.15). And vice versa, the small 12 leptonic mixing can have a natural link to the significant 12 mixing in quarks, expressed as in eq. (1.16).

One has to remark, however, that besides the AN and SN anomalies, there is another, though rather controversial, indication for the neutrino oscillations: the LSND anomaly [35]. Namely, the data collected by the LSND collaboration indicate neutrino oscillations in both the $\bar{\nu}_\mu - \bar{\nu}_e$ and $\nu_\mu - \nu_e$ channels. Although these are almost in conflict with the data of the KARMEN experiment [36], which excludes the bulk of the relevant parameter region, the following range is still allowed: $\delta m^2_{e\mu} = (0.2 - 0.6)$ eV^2 , $\sin^2 2\theta_{e\mu} = (0.4 - 4) \cdot 10^{-2}$.

If the LSND anomaly is indeed confirmed in future experiments, then three standard neutrinos ν_e, ν_μ and ν_τ will not suffice for reconciling AN and SN solutions with the parameter range required by the LSND. Since the existence of the fourth active neutrino is excluded by the LEP measurements of the invisible decay width of Z-boson, one has to introduce an extra light sterile neutrino ν_s [37]. Then several exotic textures [38] can be considered for accomodating all the data and one has also to think of the physical reasons for the existence of the light sterile neutrinos. For some older and recent works on these directions see e.g. ref. [39].

Acknowledgments

We would like to thank Alexei Smirnov and Francesco Vissani for useful discussions, and Andrea Brignole for valid observations. Z.B. thanks the high energy theory groups of the ICTP and SISSA for the hospitality in Trieste during the period when this work has been done.

Note added at replacement: a week after our paper has been submitted to the hep-ph E-print Archive, there appeared also a work by K. Hagiwara and N. Okamura (`hep-ph/9811495`) with similar considerations.

A. Diagonalization of the Yukawa matrices

Consider the Yukawa couplings of any type of fermions $f^c_i \mathbf{Y}^{ij} f_j \phi$, $f = u, d, e$, with the matrix $\mathbf{Y}$ having the Fritzsch-like form with generically complex elements:

$$\mathbf{Y} = \begin{pmatrix} 0 & A' e^{i\alpha'} & 0 \\ A e^{i\alpha} & 0 & B' e^{i\beta'} \\ 0 & B e^{i\beta} & C \end{pmatrix} \tag{A.1}$$

obeying the hierarchy $C > B, B' > A, A'$ (without loss of generality, the 33 element can be taken real). It can be brought to the diagonal form by bi-unitary transformation:

$$U'^T \mathbf{Y} U = \mathbf{Y}_D = \begin{pmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix}, \tag{A.2}$$

where $Y_3 \gg Y_2 \gg Y_1$ are the Yukawa eigenvalues for the physical fermions of three families. The unitary matrices can be parametrized as $U = PO$ and $U' = P'O'$, where the phase transformations

$$P' = \begin{pmatrix} e^{i(\pi - \alpha' + \beta)} & 0 & 0 \\ 0 & e^{-i\beta'} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad P = \begin{pmatrix} e^{i(\pi - \alpha + \beta')} & 0 & 0 \\ 0 & e^{-i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A.3}$$

bring $\mathbf{Y}$ to the real form

$$P' \mathbf{Y} P = \tilde{\mathbf{Y}} = \begin{pmatrix} 0 & -A' & 0 \\ -A & 0 & B' \\ 0 & B & C \end{pmatrix}, \tag{A.4}$$

which further can be diagonalized by the bi-orthogonal transformation $O'^T \tilde{\mathbf{Y}} O$, with the matrix O rotating the left states f parametrized as

$$O = O_{23} O_{13} O_{12} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A.5}$$

and analogously $O' = O'_{23} O'_{13} O'_{12}$ for the right states f^c. Notice, that (A.5) gives the matrix O immediately in the standard parametrization (1.5) with $\delta = \pi$.

Let us compute now these rotation angles, using the fact that $O^T \tilde{\mathbf{Y}}^T \tilde{\mathbf{Y}} O = O'^T \tilde{\mathbf{Y}} \tilde{\mathbf{Y}}^T O' = \mathbf{Y}_D^2$. For the sake of accuracy, below we maintain the corrections of the order $\varepsilon \sim Y_1/Y_2$, Y_2/Y_3, but neglect the $\mathcal{O}(\varepsilon^2)$ ones ($\sim Y_1/Y_3$, $(Y_2/Y_3)^2$ etc.). As for the elements in (A.4), in first approximation we estimate that

$$C \sim Y_3\,, \qquad BB' \sim Y_2 Y_3\,, \qquad AA' \sim Y_1 Y_2\,, \tag{A.6}$$

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so that $C^2 : BB' : AA' \sim 1 : \varepsilon : \varepsilon^3$. We assume that B and B' can be substantially different, i.e. the asymmetry parameter $b = B/B'$ can be large. In particular, B can be $\sim C$, in which case $B' \sim \varepsilon C$ and thus $b \sim 1/\varepsilon$. As for A and A', we assume that they have no big asymmetry, and thus $A \sim A' \sim \varepsilon^{3/2} C$.

We start with the 23 rotation to diagonalize the lower 23 block of $\tilde{\mathbf{Y}}$:

$$O_{23}'^T \mathbf{Y}^{(0)} O_{23} = \mathbf{Y}^{(1)} = \begin{pmatrix} 0 & -c_{23}A' & -s_{23}A' \\ -c_{23}'A & -y_2 & 0 \\ -s_{23}'A & 0 & y_3 \end{pmatrix} . \tag{A.7}$$

Then we have

$$\begin{aligned} BB' &= \frac{1}{b} B^2 = y_2 y_3 \,, \\ C^2 &= y_3^2 + y_2^2 - (B^2 + B'^2) = y_3^2 \left[1 - (b + b^{-1}) \frac{y_2}{y_3} + \frac{y_2^2}{y_3^2} \right] , \end{aligned} \tag{A.8}$$

where $b = B/B'$, and for the left (f) and right (f') rotation angles we get respectively:

$$\begin{aligned} \tan 2\theta_{23} &= \frac{2BC}{C^2 - B^2 + B'^2} = 2\sqrt{\frac{by_2}{y_3}} \frac{\left[1 - (b + b^{-1})\frac{y_2}{y_3}\right]^{1/2}}{1 - 2b\frac{y_2}{y_3}} \,, \\ \tan 2\theta_{23}' &= \frac{2B'C}{C^2 + B^2 - B'^2} = 2\sqrt{\frac{y_2}{by_3}} \frac{\left[1 - (b + b^{-1})\frac{y_2}{y_3}\right]^{1/2}}{1 - 2b^{-1}\frac{y_2}{y_3}} \,. \end{aligned} \tag{A.9}$$

Thus, the expressions for θ_{23} and θ_{23}' are obtained from each other by changing $b \to b^{-1}$. The diagonal elements $y_{2,3}$, up to small corrections detected below, coincide with the Yukawa eigenvalues $Y_{2,3}$ in (A.2), $y_{2,3} \simeq Y_{2,3}$. Therefore, with increasing b from 1 to $\sim \varepsilon^{-1}$, s_{23} increases from $\sqrt{\varepsilon}$ to 1, while at the same s_{23}' decreases from $\sqrt{\varepsilon}$ to $\sqrt{\varepsilon/b} \sim \varepsilon$, so that we always have $s_{23}s_{23}' \sim \varepsilon$.

As a next step, we rotate out the 13 block of $\mathbf{Y}^{(1)}$:

$$O_{13}'^T \mathbf{Y}^{(1)} O_{13} = \mathbf{Y}^{(2)} = \begin{pmatrix} -\Delta y_1 & -c_{13}'c_{23}A' & 0 \\ -c_{13}c_{23}'A & -y_2 & s_{13}c_{23}'A \\ 0 & s_{13}'c_{23}A' & y_3 + \Delta y_3 \end{pmatrix} , \tag{A.10}$$

where the angles θ_{13} and θ_{13}' are small:

$$s_{13} = s_{23}' \frac{A}{y_3} \,, \qquad s_{13}' = s_{23} \frac{A'}{y_3} \tag{A.11}$$

so that $s_{13}s_{13}' \sim \varepsilon^4$, and

$$\Delta y_1 = s_{23}s_{23}' \frac{AA'}{y_3} \sim \varepsilon^2 Y_1 \,, \qquad \Delta y_3 = \frac{1}{2y_3}(s_{23}^2 A'^2 + s_{23}'^2 A^2) < \varepsilon^3 Y_3 \,. \tag{A.12}$$

Therefore, with a great precision, $c_{13} = c_{13}' = 1$, and $\Delta y_{1,3}$ are negligible.

Finally, we bring the 12 block of $\mathbf{Y}^{(2)}$ to the diagonal form, which step practically accomplishes the diagonalization procedure, since $\mathbf{Y}^{(3)} = \mathrm{Diag}(1,-1,1) \times \mathbf{Y}_D$ with a very good precission:

$$O_{12}'^T \mathbf{Y}^{(2)} O_{12} = \mathbf{Y}^{(3)} = \begin{pmatrix} y_1 & 0 & s_{12}s_{13}c_{23}'A \\ 0 & -y_2+\Delta y_2 & c_{12}s_{13}c_{23}'A \\ s_{12}'s_{13}'c_{23}A' & c_{12}'s_{13}'c_{23}A' & y_3 \end{pmatrix} \approx \begin{pmatrix} Y_1 & 0 & 0 \\ 0 & -Y_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix}. \tag{A.13}$$

Therefore, up to corrections $\mathcal{O}(\varepsilon^2)$, we have $y_3 = Y_3$, $y_2 - \Delta y_2 = Y_2$ and $y_1 = Y_1$. The next series of subsequent rotations would bring at most $\mathcal{O}(\varepsilon^2)$ corrections to the mixing angles and Yukawa eigenvalues. Furthemore, we obtain the following relations:

$$\begin{aligned} c_{23}c_{23}'AA' &= \frac{1}{a}(c_{23}'A)^2 = Y_1Y_2\,, \\ y_2^2 &= Y_2^2 + Y_1^2 - (c_{23}^2A'^2 + c_{23}'^2A^2) = Y_2^2\left[1-(a+a^{-1})\frac{Y_1}{Y_2} + \frac{Y_1^2}{Y_2^2}\right], \end{aligned} \tag{A.14}$$

where $a = (Ac_{23}'/A'c_{23})$. Now Y_2 and Y_1 practically coincide with the light generation of the Yukawa constants: e.g. $Y_{1,2} = Y_{e,\mu}$ for the charged leptons. Thus we have

$$y_2 = Y_2 - \frac{1}{2}(a+a^{-1})Y_1 = Y_2 - Y_1\left[1+\frac{1}{2}(1-a)^2+\cdots\right], \tag{A.15}$$

since for $a \approx 1$, we have $a + a^{-1} = 2 + (a-1)^2 + \cdots$, and hence $y_2 \approx Y_2 - Y_1$. In conclusion th expressions for the mixing angles are the following:

$$\begin{aligned} \tan 2\theta_{23} &= 2\sqrt{b}\sqrt{\frac{Y_2-Y_1}{Y_3}}\,\frac{\left[1-(b+b^{-1})\frac{Y_2-Y_1}{Y_3}\right]^{1/2}}{1-2b\frac{Y_2-Y_1}{Y_3}}\,, \\ \tan 2\theta_{12} &= 2\sqrt{a}\sqrt{\frac{Y_1}{Y_2}}\,\frac{\left[1-(a+a^{-1})\frac{Y_1}{Y_2}\right]^{1/2}}{1-2a\frac{Y_1}{Y_2}}\,, \\ \sin\theta_{13} &= s_{23}'\frac{A}{Y_3} = \sqrt{a}\frac{s_{23}'}{c_{23}'}\frac{\sqrt{Y_1Y_2}}{Y_3}\,, \end{aligned} \tag{A.16}$$

while the expressions for θ_{23}', θ_{12}' and θ_{13}' are obtained from the above ones by changing $b \to 1/b$, $a \to 1/a$ and $s_{23}' \to s_{23}$ ($c_{23}' \to c_{23}$).

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